Noncommutative rational Pólya series

Daniel Smertnig

Department of Mathematical Sciences, Graz University, Austria.

a joint work with

Jason Bell.

Department of Pure Mathematics, University of Waterloo, Canada

Abstract

A noncommutative (formal, multivariate) power series F is rational if it can be obtained from noncommutative polynomials via the natural operations addition, multiplication, and the operation $1/(1-G)$. In the univariate case, this holds if and only if F is the power series expansion of a rational function at 0. A rational series with coefficients in a field K is a Plya series if all nonzero coefficients are contained in a finitely generated subgroup of $K^\times$.

Generalizing results of Pólya (1921), Benzaghou (1970), and Bzivin (1987) for the univariate case, we characterize multivariate rational Plya series, thereby confirming a conjecture of Reutenauer from 1979.